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Fuzzy Entropy Based MR Image Segmentation Using Particle Swarm Optimization

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Abstract— An image segmentation technique based on fuzzy entropy is applied for MR brain images to detect a brain tumor is presented in this paper. The proposed method performs image segmentation based on adaptive thresholding of the input MR images. The image is classified into two membership functions, whose member functions of the fuzzy region are Z-function and S-function. The optimal parameters of these membership functions are determined using Particle Swarm Optimization algorithm by maximizing the fuzzy entropy. Through a number of examples, the performance is compared with those using existing entropy-based object segmentation approaches and the superiority of the proposed method is demonstrated. The experimental results are compared with the exhaustive method and Otsu segmentation technique; the results show the proposed fuzzy entropy method integrated with PSO achieves maximum entropy with proper segmentation of infected areas and with minimum computational time.

Index Terms — Fuzzy entropy, image segmentation, threshold; particle swarm optimization

1 Introduction

THE goal of image segmentation is to extract meaningful objects from an input image. Image segmentation, with wide recognized significance, is one of the most difficult low-level image analysis tasks, as well as the bottle-neck of the development of image processing technology. All the subsequent tasks, including feature extraction, model matching and object recognition rely heavily on the quality of the image segmentation process.

Thresholding is undoubtedly one of the most popular segmentation approaches for the sake of its simplicity. It is based on the assumption that the objects can be distinguished by their gray levels. It is an important issue to find a correct gray level threshold that can separate different objects or separate objects from background. However, the automatic selection of a robust, optimum threshold has remained a challenge in image segmentation. An early review of thresholding methods was reported [1],[2]. Pun (1980) described a method that maximizes the upper bound of the posteriori entropy derived from the histogram. Wong and Sahoo's method determines the optimum threshold by maximizing the posteriori entropy subject to certain inequality constraints that characterize the uniformity and shape of the segmented regions. Pal and Pal (1989, 1991) developed another entropy-based method by considering the joint probability distribution of the neighbouring pixels, which they further modified with a new definition of entropy [3],[4]. The method of Kapur et al. (1985) selects the optimum threshold by maximizing the sum of entropies of the segmented regions [5]. A similar approach was reported by Abutaleb (1989), which maximizes the 2D entropy [6]. Brink's method (Brink, 1994) maximizes the sum of the entropies computed from two autocorrelation functions of the thresholded image histograms [7]. A fuzzy entropy is a function on fuzzy sets that becomes smaller when the sharpness of its argument fuzzy set is improved. The notion of entropy, in the theory of fuzzy sets, was first introduced by Luca and Termini (1972) [8]. There have been numerous applications of fuzzy entropies in image segmentation. Cheng et al. (1998) proposed fuzzy homogeneity vectors to handle the grayness and spatial uncertainties among pixels, and to perform multi-level thresholding [9]. Cheng et al. (1999) presented a thresholding approach by performing fuzzy partition on a two-dimensional histogram based on fuzzy relation and maximum fuzzy entropy principle [10]. Cheng et al. (2000) defined a new approach to fuzzy entropy, used it to select the fuzzy region of membership function automatically so that an image is able to be transformed into fuzzy domain with maximum fuzzy entropy, and implemented genetic algorithm to find the optimal combination of the fuzzy parameters. Cheng has employed the proposed approach to perform image enhancement and thresholding, and obtained satisfactory results [11]. Zhao et al. (2001) presented an entropy function by the fuzzy c-partition (FP) and the probability partition (PP) which was used to measure the compatibility between the PP and the FP. Zhao used the simplest function, that is monotonic, to approximate the memberships of the bright, the dark and the medium and derived a necessary condition of the entropy function arriving at a maximum.

In this paper, a new bi-level thresholding method for image segmentation is proposed. The paper defines a new fuzzy entropy through probability analysis, fuzzy partition and entropy theory. The image is partitioned into two parts, namely dark, and white part, whose member functions of the fuzzy region are Z-function and S-function respectively. The width and attribute of the fuzzy region can be decided by maximum fuzzy entropy, in turn the thresholds can be decided by the fuzzy parameters. For getting optimal thresholds, we must find the optimal combination of all the fuzzy parameters. Thus, the segmentation problem can be formulated as an optimal problem. The fuzzy entropy of the image has been chosen as the objective function.

We have reviewed many techniques, commonly used for function optimization, in the view of determining their usefulness for this particular task. PSO is able to overcome many of the defects in other optimization techniques such as exhaustive techniques, calculus-based techniques, partial knowledge (hill climbing, beam search, best first, branch and bound, dynamic programming, A*), knowledge based techniques (production rule systems, heuristic methods). They search from a population of individuals (search points), which make them ideal candidates for parallel architecture implementation, and are far more efficient than exhaustive techniques. PSO has been used to solve various problems in computer vision, including image segmentation. In this paper, we propose using PSO in finding the optimal combination of all the fuzzy parameters efficiently. The experiment results show that the proposed method gives better performance.

1.1 Image As A Fuzzy Event

Consider an image A of size MxN with L gray levels ranging from r_o to r_{L-1} i.e, $G = \{r_0, r_1, r_2, ..., r_{L-1}\}$. Let a_{ij} denote the gray level of the image A at the (i, j) th pixel. The histogram of the image is denoted as h_i and is defined as

$$h_i = \frac{n_i}{MxN}, i = 0,1,...,L-1 \tag{1}$$

where, n_i denotes the number of occurrences of gray levels in A.

A probability space based fuzzy event can be modeled for an image. According to fuzzy set theory, the image A can be transformed into an array of fuzzy singletons S by a membership function.

$$S = \{\mu_A(a_{ij}), i = 1,2,...M; j = 1,2,...N\} \tag{2}$$

Then, the degree of some properties of the image such as brightness, darkness, etc. possessed by the (i, j) th pixel is denoted by the membership function $\mu_A(a_{ij})$ of the fuzzy set, $A \in G$. In fuzzy set notation, A can be written as

$$A = \mu_A(r_1)/r_1 + \mu_A(r_2)/r_2 + ...\mu_A(r_i)/r_i \tag{3}$$

or

$$A = \sum_{r_i \in G} \mu_A(r_i)/r_i \tag{4}$$

Here "+" indicates union.

The equation to obtain the probability of A is given as

$$P(A) = \sum_{i=0}^{L-1} \mu_A(r_i)\Pr(r_i) \tag{5}$$

and the equation corresponding to conditional probability tends to be,

$$P[\{r_i\} \mid A] = \mu_A(r_i)h_i / P(A) \tag{6}$$

1.2 Probability Partition Based on Maximum Fuzzy Entropy

Fuzzy entropy describes the fuzziness of a fuzzy set. It is a measure of the uncertainty of a fuzzy set. The domain of the image be given as Z

$$Z = \{(i, j) : i = 0,1,2,..., M-1; j = 0,1,2,..., N-1\} \tag{7}$$

where M and N are two positive integers. If the gray level value of the image at the pixel (x, y) is $A(x, y)$, then

$$Z_k = \{(x, y) : A(x, y) = k, (x, y) \in D\}, k = 0,1,..., L-1 \tag{8}$$

Let the threshold of the image A be T that segments an image into its target and background. The domain Z of the original image can be classified into two parts, F_d and F_b, which is composed of pixels with low gray levels and high gray levels, respectively. An unknown probabilistic partition of Z denoted as $\prod_2 = \{F_d, F_b\}$ describes its probability distribution as

$$p_d = P(F_d) \tag{9}$$

$$p_b = P(F_b) \tag{10}$$

For an image with 256 gray levels, μ_b and μ_d indicates the membership functions that corresponds to the bright and dark pixels. Let a, b and c be the three parameters of the membership function, which means that the threshold T depends on a, b and c. Consider

$$Z_{kd} = \{(x, y) : I(x, y) \le T, (x, y) \in Z_k\} \tag{11}$$

$$Z_{kb} = \{(x, y) : I(x, y) > T, (x, y) \in Z_k\} \tag{12}$$

for each $k = 0,1,...,255$.

Then the following equations hold:

$$p_{kd} = P(Z_{kd}) = p_k * p_{d|k} \tag{13}$$

$$p_{kb} = P(Z_{kb}) = p_k * p_{b|k} \tag{14}$$

The conditional probability of a pixel, obviously set as $p_{d|k}$ and $p_{b|k}$ is categorized into the class 'dark 'and class 'bright', with the constraint that the pixel belongs to D_K with

$$p_{d|k} + p_{b|k} = 1, (k = 0,1,...,255) \tag{15}$$

The grade of pixels classified into class 'dark' and class 'bright' having the gray level value k be equal to its conditional probability $p_{d|k}$, $p_{b|k}$, respectively [3], [11], [12], [14].

The equations for probability p_d and p_b hold as follows:

$$p_d = \sum_{k=0}^{255} p_k * p_{d|k} = \sum_{k=0}^{255} p_k * \mu_d(k) \tag{16}$$

$$p_b = \sum_{k=0}^{255} p_k * p_{b|k} = \sum_{k=0}^{255} p_k * \mu_b(k) \tag{17}$$

The two membership functions, S - function and Z -function are applied for calculating the fuzzy entropy function which is shown in Figure 1. Here $Z(k,a,b,c)$ - function denotes the membership function $\mu_d(k)$ of the class 'dark' and $S(k,a,b,c)$ - function denotes the membership function $\mu_b(k)$ of the class 'bright' [9],[10].

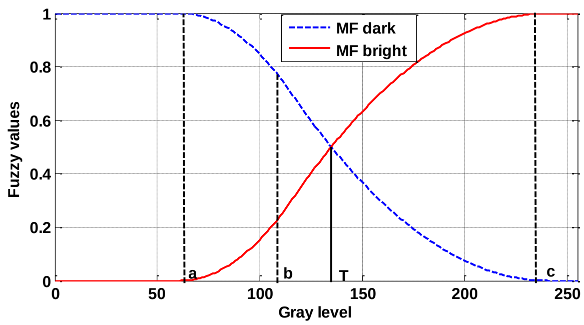

Fig. 1. Membership function graph showing the intersection of Z membership function- $\mu_d(k)$ and S membership function- $\mu_b(k)$ at T

The fuzzy parameters a, b, c satisfy the constraint: $0 \le a \le b \le c \le 255$. The equation (18) shows the membership function of *Z (k, a, b, c)*. The equation (19) shows the membership function of *S (k, a, b, c)*.

$$\mu_d(k) = \begin{cases} 1, & k \le a \\ 1 - \dfrac{(k-a)^2}{(c-a)*(b-a)}, & a < k \le b \\ \dfrac{(k-c)^2}{(c-a)*(c-b)}, & b < k \le c \\ 0, & k > c \end{cases} \tag{18}$$

$$\mu_b(k) = \begin{cases} 0, & k \le a \\ \dfrac{(k-a)^2}{(c-a)*(b-a)}, & a < k \le b \\ 1 - \dfrac{(k-c)^2}{(c-a)*(c-b)}, & b < k \le c \\ 1, & k > c \end{cases} \tag{19}$$

Fuzzy entropy function for dark class H_d is calculated based on Equation (20) and for bright class, H_b is calculated based on Equation (21) as shown below:

$$H_d = -\sum_{k=0}^{255} \frac{p_k * \mu_d(k)}{p_d} * \log\left(\frac{p_k * \mu_d(k)}{p_d}\right) \tag{20}$$

$$H_b = -\sum_{k=0}^{255} \frac{p_k * \mu_b(k)}{p_b} * \log\left(\frac{p_k * \mu_b(k)}{p_b}\right) \tag{21}$$

Then, the total fuzzy entropy function $H(a,b,c)$ is given as

$$H(a,b,c) = H_d + H_b \tag{22}$$

This total fuzzy entropy depends on the fuzzy parameters - a, b, c. The combination of these three parameters is chosen such that the total fuzzy entropy $H(a,b,c)$ attains a maximum value. The equation to segment the image into two classes using appropriate threshold is as follows:

$$\mu_d(T) = \mu_b(T) = 0.5 \tag{23}$$

From the Figure 1, it is evident that threshold T is the point of intersection of $\mu_d(k)$ and $\mu_b(k)$ curve. The solution to derive T can be obtained from the Equation (24). Hence the threshold T can be easily formed by:

$$T = \begin{cases} a + \sqrt{(c-a)*(b-a)/2}, & (a+c)/2 \le b \le c \\ c - \sqrt{(c-a)*(c-b)/2}, & a \le b \le (a+c)/2 \end{cases} \tag{24}$$

2 PARTICLE SWARM OPTIMIZATION

Particle Swarm Optimization (PSO) is a new evolutionary computing method that was developed by Kennedy and Eberhart in 1995 through the simulation of simplified social models of bird flocks [15]. Due to its excellent performance, PSO has become one of the hotspots in evolutionary computing research and has been used in many applications such as function optimization, neural network training, and fuzzy control systems in recent years. Image segmentation can be defined as the technique of dividing an image into disjoint homogeneous regions that usually contain similar objects of interest. And it is an important step in automatic image analysis and interpretation. However, due to the uncertainty and complexity of images encountered in actual applications, it is a very difficult task that affects directly the results of subsequent tasks such as feature extraction, object detection, and recognition. Because fuzzy set theory is an effective means of researching and processing fuzziness and uncertainty, fuzzy entropy has been used in image threshold segmentation.

PSO is a population-based algorithm that uses a population of individuals to probe the best position in the search space. In PSO, the individual is called a particle, which moves with an adaptable velocity in the search space [15]. Each particle moves stochastically in the direction of its own best previous position and the whole swarm's best previous position. Suppose that the size of the swarm is N and the search space is M dimensional, then the position of the i[th] particle is presented as $X_i(x_{i1}, x_{i2}, x_{i3}, ..., x_{im})$. The velocity of this particle is presented as $V_i(v_{i1}, v_{i2}, v_{i3}, ..., v_{im})$. The best previous position of this particle is denoted as $P_i(p_{i1}, p_{i2}, p_{i3}, ..., p_{im})$ and the best previous position discovered by the whole swarm is denoted as $P_g(p_{g1}, p_{g2}, p_{g3}, ..., p_{gm})$. The particles are manipulated according to the following equations : where $1 \le m \le M$, and $rand()$ is the random number with uniform distribution $U(0,1)$; c_1 and c_2 are acceleration coefficients; ω is the inertia weight; $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum value of ω, respectively; k and $k_{\max}$ are the current iterative time and the maximum iterative time, respectively; usually Δt is unit time.

$$_{n+1}v_i^d = \omega_n v_i^d + c_1 r_{1_i}^d (pbest_i^d - {}_n x_i^d) + c_2 r_{2_i}^d (gbest^d - {}_n x_i^d) \quad (25)$$

$$_{n+1}x_i^d = {}_n x_i^d + {}_{n+1}v_i^d \quad (26)$$
$$i = 1,2,3,..., N, d = 1,2,3,..., D.$$

where, $x_i = (x_i^1, x_i^2, x_i^3, \ldots, x_i^D)$ is the position of the i^{th} particle, $pbest_i = (pbest^1, pbest^2, \ldots, pbest^D)$ is the best local best position of a particle, $gbest = (gbest^1, gbest^2, \ldots, gbest^D)$ is the global best position discovered by the entire population, $v_i = (v_i^1, v_i^2, v_i^3, \ldots, v_i^D)$ is the velocity of a particle *i*, c_1 and c_2 are the acceleration constants representing the weighting of stochastic acceleration terms that pull each particle towards *pbest* and *gbest* positions, n is the migration number, r_1 and r_2 are the random variables in the range [0,1] and ω is the inertia weight used to balance between the global and local search abilities [15].

The coefficients *c*1 and *c*2 can be expressed as follows:

$$c_1(iter) = (c_{1,\min} - c_{2,\max})\frac{iter}{iter_{\max}} + c_{1,\max}$$

$$c_2(iter) = (c_{2,\max} - c_{1,\min})\frac{iter}{iter_{\max}} + c_{2,\min}$$

where, *iter* is the current iteration number and *itermax* is the maximum iteration number. Then $_{n+1}v_i^d$ and $_{n+1}x_i^d$ should be under the constrained conditions as follows:

$$_{n+1}v_i^d = \begin{cases} _{n+1}v_i^d, & -v_{\max} \le {}_{n+1}v_i^d \le v_{\max} \\ v_{\max}, & _{n+1}v_i^d > v_{\max} \\ -v_{\max}, & _{n+1}v_i^d < -v_{\max} \end{cases} \quad (27)$$

$$_{n+1}x_i^d = \begin{cases} _{n+1}x_i^d, & x_{\min} \le {}_{n+1}x_i^d \le x_{\max} \\ x_{init}, & _{n+1}x_i^d > x_{\max} \\ x_{init}, & _{n+1}x_i^d < x_{\min} \end{cases} \quad (28)$$

3 PROPOSED THRESHOLDING METHOD

Three fuzzy parameters (*a*, *b* and *c*) are used to design fuzzy MFs. The two membership functions are constructed by these three parameters subject to the constraint; $0 \le a \le b \le c \le 255$. These three paramerters are optimized using PSO. Since PSO uses objective function to find its optimal solution, entropy is considered as the objective function based on Equation (22). This optimization is considered as a minimization problem. Hence, the fitness function is considered as inverse of objective function. The threshold is calculated from the optimal fuzzy MFs parameters and segmentation is carried out. The procedure can be summarized as follows: Initialization of the particle swarm for the position matrix X and the velocity matrix V are given below as:

$$x_{init} = x_{\min} + (x_{\max} - x_{\min}) * rand() \quad (29)$$

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ \ldots & \ldots & \ldots \\ x_{N1} & x_{N2} & x_{N3} \end{bmatrix} \quad (30)$$

$$_n v_i^d = -v_{\max} + 2v_{\max} * rand() \quad (31)$$

$$V = \begin{bmatrix} v_{11} & v_{12} & v_{13} \\ v_{21} & v_{22} & v_{23} \\ \ldots & \ldots & \ldots \\ v_{N1} & v_{N2} & v_{N3} \end{bmatrix} \quad (32)$$

where, $x_{\max}$ and $x_{\min}$ are the maximum and minimum value of position (x)

where, $x_{\max} = L_{\max}$,

$x_{\min} = L_{\min} + 1$,

$x_{i2} - x_{i1} \ge 2$ and $x_{i3} - x_{i2} \ge 2$;

$L_{\max}$ and $L_{\min}$ are the corresponding maximum and minimum gray levels of the image.

For each particle, fitness value is calculated using the fuzzy entropy function. The evaluated current fitness values are compared with that of the fitness value of its best previous position. If the current fitness value is found to be better, then the best previous position is set as the current best position. Then compare the evaluated fitness value of each particle with the fitness value of the whole swarm's best previous position, *pbest*. If the current value is better, subsequently set the current position as the whole swarm's best previous position. Update the velocity of each particle using the Equation (25) and update the position of each particle with Equation (26), subject to constrains equations (27) and (28). The predefined maximum iterative time is the stopping criterion. If the terminating criterion is not satisfied, the PSO will search for the next best particle in the swarm. When the terminating criterion is satisfied, the threshold T is calculated based on the optimal fuzzy MF

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parameters (a,b,c) then segmentation is carried out.

4 EXPERIMENTAL RESULTS

The entire simulation was carried out using MATLAB 7.10.0 on an Intel® Core™ i5-2430M CPU @ 2.40 GHz, 4GB RAM - HP Pavilion G6 notebook computer. The PSO parameters were initialized as mentioned in Table 1.In substantiating this work, a set of images was used to evaluate the performance of the proposed algorithm as well as some of the commonly used algorithms. Each image includes distinct object and the background, and the object can be exactly distinguished from the background by some suitable threshold.

TABLE 1
PARAMETER SETTINGS FOR THE PROPOSED METHODOLOGY

Parameters		Value
Swarm size		25
Self-recognition coefficient, c1	$c_{1\,min}$	0.5
	$c_{1\,max}$	2.5
Social coefficient , c2	$c_{2\,min}$	0.5
	$c_{2\,max}$	2.5
Inertia weight, ω		1
Bird step		150

A set of magnetic resonance (MR) brain images are selected as the experimental data. MR brain images are selected to detect and segment out the abnormalities or the diseased part in the brain. The diseased portion of the MR brain image can be separated from the background by applying an appropriate threshold.

In this section, the simulation results including illustrative examples and performance evaluating tables which clearly demonstrate the merits of the proposed method are discussed. In order to validate the effectiveness of the proposed method, it is compared with exhaustive search method and Otsu's thresholding method. The simulation results of two MR brain images are shown in figures 2 - 3. The MR brain images used for comparison include a tumor in the supersellar region (Fig. 2(a)) and tumor in the left medial parietal cortex (Fig. 3(a)).

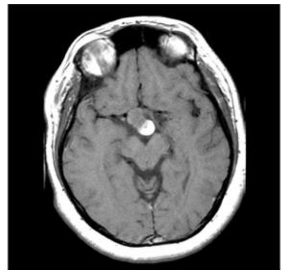

Fig. 2(a)

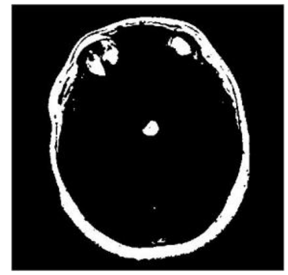

Fig. 2(a)

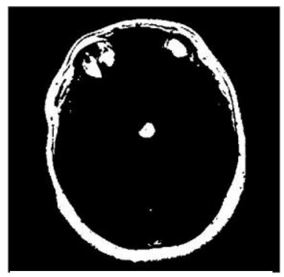

Fig. 2(a)

Fig. 2(a)

Fig. 2. Image segmentation on MR image 2 (630 x 612 x 3) (a) Original image 2; (b) thresholding result by the proposed method (T= 193.74); (c) results by exhaustive search method (T= 193.74); (d) Otsu segmentation (T=82.32)

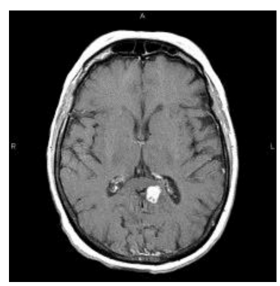

Fig. 3(a)

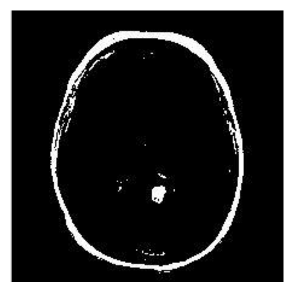

Fig. 3(b)

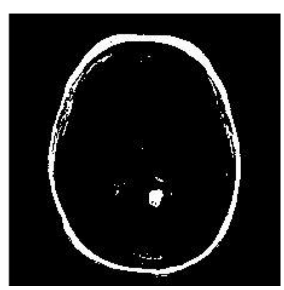

Fig. 3(c)

Fig. 3(d)

Fig. 3. Image segmentation on MR image 1(225 x 225) (a) Original image 1; (b) thresholding result by the proposed method (T= 184.82); (c) results by exhaustive search method (T= 184.82); (d) Otsu segmentation (T=74.29)

5 Conclusion

In the paper, a bi-level thresholding method for MR images based on maximum fuzzy entropy was introduced. A modification was introduced in a standard PSO in order to have better output. To analyze the performances of the proposed method, the results were compared with that of conventional thresholding methods. The results show that the proposed method obtains satisfactory performances in the segmentation experiments conducted for the test image. To ensure the optimized fuzzy parameters, the results are compared with conventional search method (enumerative search method). The proposed method is capable of finding the global optimal fuzzy membership parameters as that of the conventional search method with minimum computational time. Hence, it is concluded that fuzzy entropy based image segmentation technique using PSO is one of the effective method for thresholding MR images. This method can be integrated as an alternate to bi-level segmentation to obtain better performances in any image processing applications.

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